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13. ABSTRACT *(Maximum 200 words)*

This project concerns properties of wave propagation in nonlinear partial differential equations of hyperbolic and parabolic type. The analysis is motivated by applications in mechanics, notably the flow of granular materials, and the spread of thin liquid films on a solid substrate. The methods used involve numerical simulations and mathematical analysis, both of the numerical results, and of the underlying phenomena and pdes. In the case of liquid films, the analysis and numerical results are compared to experiments. For granular materials, the results are related to practical considerations of hopper design and instabilities in real flows.

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Nonlinear Differential Equations and Mechanics

FINAL PROGRESS REPORT

MICHAEL SHEARER

DECEMBER, 2001

U.S. ARMY RESEARCH OFFICE

GRANT NUMBER DAAG55–98–1–0128

NORTH CAROLINA STATE UNIVERSITY

A. STATEMENT OF THE PROBLEM STUDIED

This project concerns properties of wave propagation in nonlinear partial differential equations of hyperbolic and parabolic type. The analysis is motivated by applications in mechanics, notably the flow of granular materials, and the spread of thin liquid films on a solid substrate. The methods used involve numerical simulations and mathematical analysis, both of the numerical results, and of the underlying phenomena and pdes. In the case of liquid films, the analysis and numerical results are compared to experiments. For granular materials, the results are related to practical considerations of hopper design and instabilities in real flows.

B. SUMMARY OF RESEARCH RESULTS

This description of results from the project covers research in two areas:

1. Flow of granular materials.
2. Propagation of thin liquid films on a solid surface.

1. GRANULAR MATERIALS

The research into properties of granular materials concerned: (i) Steady flow in hoppers; (ii) Shear bands in dynamic equations; (iii) Discrete models to describe stress fluctuations observed experimentally.

(i) Steady flow in hoppers.

In papers [8, 9], we study steady flow in conical hoppers, a topic of considerable importance to industry. The issues here are (a) to generate a reliable numerical code that captures the structure caused by changes in hopper geometry or wall friction, and (b) to understand the engineering approximation of flow around a conical insert, used to mobilize material at the hopper wall in a container whose walls are too shallow.

The conclusion of the first study was that while stress calculations are reliable and clearly show the detailed effect of changes in the boundary, velocity calculations are very sensitive, and generally indicate a violation of energy principles in some region of the flow. This observation has raised several profound issues with modeling and analysis that are the subject of continuing research.

Flow around conical inserts is presumably quite complex, but a clever idea has been used by our industrial collaborators (Jenike and Johansson, Inc.) to approximate the flow by a self similar flow that can be found simply. This leads to a manageable criterion for the design and positioning of conical inserts. Our analysis and numerical trials establish that this idea has very limited applicability and a more comprehensive simulation of the flow is needed to properly quantify the effects of conical inserts.

(ii) Shear bands.

This part of the research focuses on a continuum model of antiplane shear. The simplest such model [1,13] assumes the material is at yield everywhere and is deforming plastically. The

PDE model is related to equations studied in connection with image enhancement, notably the Peron-Malik model. In the case of granular materials, spatial discretization of the linearly ill-posed PDE leads to a well-posed system of ODEs. Linear ill-posedness of the PDE is observed in short-time growth of oscillations in the ODE solutions. However, these saturate rapidly due to nonlinearity, and there follows a slow process of coarsening, with eventual convergence to a steady solution with a single shear band. The analysis establishes that solutions with a single shear band are stable, while those with more than one shear band are unstable. Interesting details emerged from the simulations, such as time scales over which the coarsening takes place. In particular, these results imply that in the continuum limit, the solution never evolves from the initial data. In this sense, the discrete equations are a better model of global behavior than the continuum PDE.

More recent results [14] concern more realistic models that include elasticity, including the possibility of elastic unloading. The results on stability of a shear band now depend on the size of a dimensionless parameter, the shear modulus. For large shear modulus, the results mimic those of the inelastic case, formally the limit of infinite shear modulus. However, as this modulus decreases, an eigenvalue crosses the imaginary axis and the shear band equilibrium loses stability. Subsequent behavior for smaller shear modulus (in fact the most relevant range physically) is observed numerically to be a combination of plastic deformation and elastic unloading, apparently with a periodic structure. The analysis of these solutions is still at a preliminary stage, and is the subject of an ongoing Ph.D. thesis project.

(iii) Discrete Models.

In [2,5], discrete models for stress fluctuations in strain-hardening elastoplasticity are formulated and analyzed, both numerically and mathematically. The simple model of reference [5], motivated by experiments measuring stress fluctuations, consists of a system of frictional blocks connected to a rigid rod by leaf springs. The friction law is position dependent, and gives rise to a set of phenomena that parallel the experimental observations. Numerical simulations using a discrete element model of particle interactions are carried out in [2].

2. THIN LIQUID FILMS

In a series of papers [7,10,11,12], we have explored the structure, existence and stability of waves representing driven flow of a thin liquid film up an inclined solid surface. The most striking discovery is that the well-known capillary ridge near the contact line, where fluid collects, broadens as the wave progresses, thus relieving the notorious fingering instability typically observed in driven flows. The analysis that goes into understanding this new structure involves the theory of nonstandard shock waves, the so-called undercomressive shocks [1], studied in some detail for scalar conservation laws and certain systems in earlier papers, also under this grant [3,4,6]. Mathematically, the novel feature of thin film flow is that the smoothing terms come from capillarity, i.e., surface tension, and are therefore fourth order rather than second order diffusion and third order dispersion.

Progress has also been made in understanding the motion of contact lines. There is a well known stress singularity at the contact line, but this is avoided by either including a thin precursor layer, as in the papers summarized above, or by relaxing the no-slip boundary

condition to the so-called Navier slip condition, in which a small amount of slip is allowed in a small neighborhood of the contact line. With such a condition, the problem of finding a travelling wave that terminates in a contact line has some interesting mathematics. We have constructed an asymptotic expansion for the solution, and have used the expansion to explore numerical solutions. These solutions have an interesting connection to an empirical law relating the speed of the contact line to the slope of the film at the contact. This work is in the form of a preprint and will be submitted in the next month or so.

C. PUBLICATIONS

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14. (with T.P. Witelski and D.G. Schaeffer) One dimensional solutions of an elastoplasticity model for granular materials. In preparation.

15. (with A.L. Bertozzi and R. Buckingham) The motion of thin film contact lines and the Navier slip condition. Preprint.

D. PERSONNEL SUPPORTED

Michael Shearer.

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